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Malliavin-Skorohod calculus for additive processes with applications to Finance and Insurance

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Goal of the course I

To develop a Malliavin-Skorohod type calculus for Lévy and additive processes, that extends the more known Malliavin-Skorohod calculus for Gaussian processes.

Recall that Lévy processes are processes with independent and stationary increments. And additive processes are processes with independent increments but not necessarily stationary.

In particular, first of all we will develop a non-probabilistic Malliavin-Skorohod type calculus on the abstract structure of Fock space. Later, we will identify the space of (square integrable) functionals of an additive process as a Fock space and we will give a probabilistic interpretation of Fock space operators as operators on the canonical space of an additive process.

Goal of the course II

The goal of the second part of the course is to show some selected applications of Malliavin-Skorohod calculus for Lévy or additive process to financial problems in the context of stochastic volatility jump-diffusion models.

Concretely, possible applications are pricing and hedging derivatives, computation of Greeks, implied volatility surface modeling, or pricing cumulative loss derivatives in Insurance.
A survey of the first application can be found in Vives (2007) and Vives (2013), and a survey related with the third application, in Vives (2016).

Goal of the course III

To follow the course it is useful the reader has some knowledge on Lévy processes and on Itô Stochastic Calculus in relation with Finance, like it is explained for example in the books of Lamberton-Lapeyre (2007) (second edition) and Cont and Tankov (2004).

Knowledge on Gaussian Malliavin calculus is also useful. A well-known complete reference is Nualart (2006)(second edition) and for a short and fast introduction, Nualart and Nualart (2018).

The contents of this course want to be part of the (I hope) future book:

S. Ortiz and J. Vives: *Malliavin-Skorohod calculus for additive processes and applications to Finance*.

Some basic bibliography

- D. Lamberton and B. Lapeyre (2007): *Introduction to Stochastic Calculus applied to Finance*. Second edition. Chapman-Hall/CRC.
- D. Nualart (2006): *The Malliavin Calculus and Related Topics*. Second edition. Springer.
- D. Nualart and E. Nualart (2018): *Introduction to Malliavin Calculus*. Cambridge.
- N. Privault (2009): *Stochastic Analysis in Discrete and Continuous Settings*. Springer.
- R. Cont and P. Tankov (2004): *Financial modeling with jump processes*. Chapman and Hall / CRC.
- G. Di Nunno, B. Oksendal and F. Proske (2009): *Malliavin calculus for Lévy processes with applications to Finance*. Springer.

Malliavin-Skorohod calculus in Finance I

During the last thirty years, Malliavin-Skorohod calculus has been applied to different topics in Finance and Insurance. Between them, we emphasize

a) Pricing and hedging financial derivatives
b) Computation of Greeks.
c) Analysis of the volatility surface.
d) Pricing cumulative loss derivatives.

Malliavin-Skorohod calculus in Finance II

Applications of Malliavin-Skorohod calculus to Finance have been developed during the last thirty years.

Probably the first one was due to Karatzas and Ocone (1991), where an elegant solution of the problem of pricing and hedging financial derivatives in complete markets was found using the currently so called Clark-Haussmann-Ocone formula.

A second key application appeared in 1999 with the celebrated paper of Fournié, Lasry, Lebuchoux, Lions and Touzi (1999), where the integration by parts formula was applied successfully to improve the efficiency in computing Greeks, reducing dramatically the computational cost of this type of numerical computations.

Malliavin-Skorohod calculus in Finance III

A third interesting application was developed by Alòs (2006), where Malliavin-Skorohod calculus was applied to obtain an expansion of the pricing formula under stochastic volatility diffusion models, that allows to distinguish clearly the effect of correlation in prices. This formula, an extension of the classical Hull and White formula, allows to obtain interesting results related with the shape of the implied volatility surface, see the survey Vives (2016).

Finally, Malliavin-Skorohod calculus provides an integration by parts formula that makes pricing of derivatives based on the cumulative loss process in Insurance easier. See for example Khalfallah-Hadji-Vives (2023).

Outline

The outline of the course is the following:

- Lecture 1: Malliavin-Skorohod calculus in Finance and Insurance.
- Lecture 2: Malliavin-Skorohod calculus without probability. The chaotic representation property.
- Lecture 3: Lévy and additive processes. The Lévy-Itô decomposition.
- Lecture 4: Malliavin-Skorohod calculus for additive processes.
- Lecture 5: Pricing and hedging financial derivatives under SVJ models.
- Lecture 6: Integration by parts. Pricing cumulative loss derivatives for SVJ models.